The 1-2-3 Conjecture and related problems

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Let G be a simple connected graph with at least two edges. The title conjecture states that it is possible to assign numbers from the set $\{1, 2, 3\}$ to the edges of G so that the resulting weighted graph has no pair of adjacent vertices with the same weighted degree. This innocent-looking statement has been resisting attacks for about twenty years, although some quite sophisticated methods have been tried. Many related problems were considered leading sometimes to unexpected strange territories. For example, is it possible to fill the cells of the infinite chessboard with numbers from the set $\{1, 2, 3\}$ in such a way that every square forms a non-singular matrix? Or, is it always possible to tile the space $\mathbb{R}^{\pi(n)}$ with integer translates of the *primordium*—the cube cluster corresponding to prime factorizations of the first n positive integers? I will present more of such crazy stuff during the talk.