On a Problem of Steinhaus

M.Anholcer(1), B.Bosek(2), J.Grytczuk(3), G.Gutowski(2), J.Przybyło(4), R.Pyzik(2), M.Zając(3),

(1) Poznań University of Economics and Business
(2) Jagiellonian University
(3) Warsaw University of Technology
(4) AGH University of Science and Technology

In this talk, inspired by the *17-points Problem* of Steinhaus (Problems 6 and 7 from his famous book *Sto zadań*), we discuss infinite sequences of real numbers in $[0, 1)$. For a function $f : \mathbb{N} \to \mathbb{N}$, we say that a sequence X is f-piercing if for every integer $m \geqslant 1$, the first $f(m)$ elements of X contain at least one element in every interval $\left[\frac{i}{m}, \frac{i+1}{m}\right)$ for every $i = 0, 1, \ldots, m-1$. There is a nice construction of an $\left(\frac{m}{\ln 2}\right)$-piercing sequence due to de Bruijn and Erdős which satisfies even stronger piercing properties. We are able to show that this is best possible, as there are no $(\alpha m + o(m))$-piercing sequences for $\alpha < \frac{1}{\ln 2}$. Our results allow for some new tight linear bounds for similar concepts defined for finite sequences. Ideas presented during this talk are described in full detail in our arXiv manuscript [1].

References

[1] M.Anholcer, B.Bosek, J.Grytczuk, G.Gutowski, J.Przybyło, R.Pyzik, M.Zając, On a Problem of Steinhaus, *arXiv:2111.01887*.