Generalized Turán number of double stars and other bipartite graphs in triangle-free graphs

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In a generalized Turán problem, two graphs H and F are given and the question is the maximum number of copies of H in an F-free graph of order n. In the first part of the talk, , we study the number of double stars $S_{k,l}$ in triangle-free graphs. We also study an opposite version of this question: what is the maximum number of edges and triangles in graphs with double star type restrictions, which leads us to study two questions related to the extremal number of triangles or edges in graphs with degree-sum constraints over adjacent or non-adjacent vertices.

In the second part of the paper, the double stars are replaced by arbitrary bipartite graphs. But first, a more general conjecture.

Conjecture. (Lidický, Murphy) Let G be a graph and let $r > \chi(G)$ be an integer. Then there exist integers $n_1, n_2, \ldots, n_{r-1}$ such that $n_1 + n_2 + \cdots + n_{r-1} = n$ and we have

$$ex(n, G, K_r) = G(K_{n_1,n_2,\ldots,n_{r-1}}).$$

Actually, there is a counterexample to the Conjecture for every $r \geqslant 3$. The case $r = 3$ is specially interesting. We found some extra conditions what make the conjecture true. But the exact condition is still not clear.

The talk is based on a paper joint with R. Wang, S. Woolfson, and another one joint with A. Grzesik, N. Salia, C. Tompkins