The longest CT-paths in 4-regular plane graphs

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Let G be a 4-regular graph with prescribed rotation system and let e_1, e_2, e_3, e_4 be edges incident with a vertex v in that order. The pairs e_1, e_3 and e_2, e_4 are called CT-adjacent in G. A CT-path (CT-trail) is a path (trail) in which every two consecutive edges are CT-adjacent. Simple 4-regular plane graphs consisting of a single closed CT-trail are called knots; if every closed CT-trail of a simple 4-regular plane graph is a CT-cycle, then the graph is called Grötzsch-Sachs graph.

In this talk, we show that the longest CT-path in an n-vertex knot has at most $n-2$ vertices, and give construction of knot with longest CT-path with that number of vertices for every $n \geqslant 8$; also we prove that the longest CT-path in an n-vertex Grötzsch-Sachs graph has at most $\frac{2n}{3}$ vertices. Next, we show that there exists infinitely many simple 4-regular plane graphs whose longest CT-paths contain just eight vertices; we conjecture that, apart of the single exception, all graphs with longest 8-vertex paths are Grötzsch-Sachs graphs. In addition, we provide an analogous construction yielding knots with longest 16-vertex paths. In the case when the longest CT-path has less than eight vertices, we pose a conjecture (supported by computer simulations generating the list of feasible graphs) that there is only finitely many corresponding 4-regular plane graphs; we have confirmed its validity for longest CT-paths on four and five vertices.