Homology of graphs and path complexes

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A *path complex* P on a finite set V is a collection of finite sequences of points $i_0 \dots i_n$ from V such that if path v belongs to P then truncated paths $i_1 \dots i_n$ and $i_0 \dots i_{n-1}$ are also in P [1]. Any poset $(V, \leqslant)$ determines naturally a path complex P with paths that are given by sequences $i_0 \dots i_n$ with $i_{k-1} < i_k$ for $k = 1, \dots, n$. Any simplicial complex S determines naturally a path complex $P(S)$ consisting of the sequences of simplexes $\sigma_0 \dots \sigma_n$ such that $\sigma_{k-1} \subsetneqq \sigma_k$. However, the main motivation for considering path complexes comes from the graph theory. Any (di)graph naturally defines a path complex where allowed paths go along edges (arrows). For any path complex P we can define a *path homology groups*. It follows from this definition that the simplicial homology is a particular case of the path homology. We present the path homology theory for various categories of graphs and describe its relations to Eilenberg-Steenrod axiomatic in the classical algebraic topology [1]. We give examples of application to colored digraphs and graphs [2, 3].

References

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