Light edges in embedded graphs with minimum degree 2

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The weight of an edge e is the degree-sum of its end-vertices. An edge $e = uv$ is of type (i, j) if $deg(u) \leqslant i$ and $deg(v) \leqslant j$. Kotzig proved that every 3-connected plane graph contains an edge of weight at most 13. Ivančo described bounds for weights of edges in the class of graphs embeddable on the orientable surfaces with higher genus. Jendroľ and Tuhársky investigated the weight of edges in the class of graphs embeddable on the nonorientable surfaces with higher genus. Later Jendroľ, Tuhársky and Voss described exact types of edges in large embedded maps with minimum degree 3.

In the talk we consider connected graphs on n vertices, minimum degree two, minimum face degree ρ, embedded on a surface $\mathcal{S}$ with non-positive Euler characteristic. We can prove that every such graph contains an edge of type

- $(2, \infty)$, $(3, 12)$, $(4, 8)$ or $(6, 6)$ if $\rho = 3$ and $n > 24|\chi(\mathcal{S})|$,
- $(2, \infty)$, $(3, 6)$ or $(4, 4)$ if $\rho = 4$ and $n > 20|\chi(\mathcal{S})|$,
- $(2, 6)$ or $(3, 3)$ if $\rho \in \{5, 6\}$ and $n > 27|\chi(\mathcal{S})|$,
- $(2, 4)$ if $\rho \in \{7, 8\}$ and $n > 14|\chi(\mathcal{S})|$,
- $(2, 3)$ if $\rho \in \{9, 10, 11, 12\}$ and $n > 15|\chi(\mathcal{S})|$,
- $(2, 2)$ if $\rho \geqslant 13$ and $n > 35|\chi(\mathcal{S})|$.

We will also discuss the quality of our results.