Distinguishing regular graphs

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Call an edge colouring of a graph G distinguishing if the only automorphism of G that preserves the colouring is the identity. The distinguishing index of graph G is the least number of colours in a distinguishing edge colouring of G, and it is denoted by $D'(G)$. Lehner, Pilśniak and Stawiski proved that the distinguishing index of every locally finite (infinite or finite) connected regular graph except K_2 satisfies $D'(G) \leqslant 3$. Grech and Kisielewicz extended this result to locally finite connected graphs for which the minimum degree is at least half the maximum degree. Lehner, Pilśniak and Stawiski conjectured that $D'(G) \leqslant 2$ for every locally finite connected regular graph on at least seven vertices. We prove this conjecture.

Pilśniak proved that the distinguishing index of finite traceable graph of order at least 7 is at most 2. It follows from our result that vertex-transitive graphs satisfy the same bound. Therefore, this may be seen as a support for the well-known Lovász Conjecture stating that every finite connected vertex transitive graph is traceable.

References

[1] M. Grech, A. Kisielewicz, A class of graphs with distinguishing index $D' \leqslant 3$, arXiv:2107.09449, 2021.

[2] F. Lehner, M. Pilśniak, M. Stawiski, A bound for the distinguishing index of regular graphs, *European J. Combin.* 89 2020, 103145.