Breaking symmetry with two colors

Thomas Tucker[1]

[1] Colgate University, Hamilton NY

Given a group A acting on a set X, an assignment of colors to the elements of X is *distinguishing* if the only element of A preserving the coloring fixes all $x \in X$. The least number of colors needed is the *distinguishing number* $D(A, X)$. If $A = Aut(G), X = V(G)$ for the graph G, then $D(A, X) = D(G)$ is called the distinguishing number (or asymmetric coloring number $ACN(G)$); if instead we use $X = E(G)$, we have the distinguishing index $D'(G)$. Albertson and Collins (1996) investigated the activity on distinguishing graphs, but, unbeknownst to them, the idea extends back to Babai (1977) for trees and Gluck(1983), Cameron et al (1984) for finite permutation groups. This talk surveys the principal results on distinguishing graphs and maps, focussing on classification of situations where $D(A, X) = 2$. Generally, if every non-identity $a \in A$ moves enough $x \in X$, then $D(A, X) = 2$. For locally finite, infinite graphs it was conjectured 10 years ago that "enough" is infinity. This Infinite Motion Conjecture has been proved in the last year by Babai.