Constructing sparsest ℓ-hamiltonian saturated k-uniform hypergraphs for a wide range of ℓ

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Given $k \geqslant 3$ and $1 \leqslant \ell < k$, an (ℓ, k)-cycle is one in which consecutive edges, each of size k, overlap in exactly ℓ vertices. We study the smallest number of edges in k-uniform n-vertex hypergraphs which do not contain hamiltonian (ℓ, k)-cycles, but once a new edge is added, such a cycle is promptly created. It has been conjectured [1, 2] that this number is of order n^ℓ and confirmed [2, 3] for $\ell \in \{1, k/2, k-1\}$, as well as for the upper range $0.8k \leqslant \ell \leqslant k-1$. Here we extend the validity of this conjecture to the lower-middle range $(k-1)/3 \leqslant \ell < k/2$.

References

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