
AGH
IX
CRACOW CONFERENCE ON GRAPH THEORY
June 5 - 10, 2022, Rytro, Poland
During the conference we shall celebrate 70th Mariusz Woźniak's birthday.
Invited speakers:
László Babai University of Chicago, USA
Jarosław Grytczuk Warsaw University of Technology, Poland
Michael Henning University of Johannesburg, South Africa
Daniel Kráľ Masaryk University in Brno, Czech Republic, and University of Warwick, United Kingdom
Michael Molloy University of Toronto, Canada
Janos Pach Rényi Institute of Mathematics, Hungary, and EPFL, Lausanne, Switzerland
Thomas Tucker Colgate University, Hamilton, NY, USA
25 years AGH Faculty of Applied Mathematics
The conference is sponsored by the Ministry of Education and Science
Scientific Organizing Committee:
Rafał Kalinowski, Monika Pilśniak, Jakub Przybyło
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I Tucker's conjecture on asymmetric 2-colorings of locally finite graphs

László Babai

University of Chicago

The *degree* of a permutation is the number of elements it does not fix. The *minimal degree* of a permutation group G is the minimum of the degrees of the non-identity elements of G. The *motion* of a structure X (such as a graph) is the minimal degree of its automorphism group, $\mathrm{Aut}(X)$. An *asymmetric coloring* (AC) for a permutation group G acting on a set Ω is a coloring of Ω that is not fixed by any non-identity element of G. An asymmetric 2-coloring (A2C) is an AC with 2 colors. An AC (A2C) for a structure X is an AC (A2C, resp.) for $\mathrm{Aut}(X)$.

Tom Tucker proposed the following "Infinite Motion Conjecture" in 2011: *Every connected locally finite graph with infinite motion admits an A2C.*

In 2018, Florian Lehner, Monika Pilśniak, and Marcin Stawiski confirmed the conjecture for graphs of degree $\leqslant 5$. In 2021 we confirmed the conjecture in full generality. We shall sketch some of the ingredients of the proof.

The problem reduces to the analogous question for the *inverse limit of a sequence of finite permutation groups with disjoint domains.* The latter reduces to questions about finite groups, and results by Gluck (1983) and Seress (1997) about A2Cs of primitive permutation groups become relevant. As an illustration, here is the central technical lemma.

Lemma. *Let G be a permutation group acting on a finite set Ω. Let $\varphi : G \twoheadrightarrow T$ be an epimorphism onto a nonabelian simple group T. Then there exists a 2-coloring γ of Ω such that $\varphi(G_\gamma)$ is a proper subgroup of T, where G_γ denotes the subgroup of G that fixes γ.*

The following open question addresses the local cost of an A2C.

Question. Does every connected locally finite *rooted* graph with infinite motion admit a red/blue A2C such that on every sphere about the root, only a bounded number of vertices are colored red?

II The 1-2-3 Conjecture and related problems

Jarosław Grytczuk

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Let G be a simple connected graph with at least two edges. The title conjecture states that it is possible to assign numbers from the set $\{1, 2, 3\}$ to the edges of G so that the resulting weighted graph has no pair of adjacent vertices with the same weighted degree. This innocent-looking statement has been resisting attacks for about twenty years, although some quite sophisticated methods have been tried. Many related problems were considered leading sometimes to unexpected strange territories. For example, is it possible to fill the cells of the infinite chessboard with numbers from the set $\{1, 2, 3\}$ in such a way that every square forms a non-singular matrix? Or, is it always possible to tile the space $\mathbb{R}^{\pi(n)}$ with integer translates of the *primordium*—the cube cluster corresponding to prime factorizations of the first n positive integers? I will present more of such crazy stuff during the talk.

III Graphs with given maximum degree and smallest possible matching number

Michael A. Henning

Department of Mathematics and Applied Mathematics
University of Johannesburg
Auckland Park, 2006 South Africa

In this talk, we present tight lower bounds on the matching number of a graph with given maximum degree in terms of its order and size, and we characterize the graphs achieving equality in these bounds. We also discuss tight lower bounds on the matching number of a graph with given maximum degree and edge-connectivity in terms of its order and size.

IV Quasirandom and common combinatorial structures

Daniel Král'

Masaryk University, Brno, Czech Republic

A combinatorial structure is said to be quasirandom if it resembles a random structure in a certain robust sense. The notion of quasirandom graphs, developed in the work of Rödl, Thomason, Chung, Graham and Wilson in 1980s, is particularly robust as several different properties of truly random graphs, e.g., subgraph density, edge distribution and spectral properties, are satisfied by a large graph if and only if one of them is. A closely related notion is the notion of common graphs, which are graphs whose number of monochromatic copies is minimized by the (quasi)random coloring of a host complete graph.

We will discuss quasirandom properties of various combinatorial structures and present several recent results obtained using analytic tools of the theory of combinatorial limits. We will then present some recent results on common and locally common graphs, in particular, we show that there exists common connected graphs with arbitrary large chromatic number, whose existence was an open problem for more than 20 years. At the end of the talk, we will mention an extension of the notion of common graphs in the algebraic setting.

The talk is based on results obtained with different groups of collaborators, including Timothy F. N. Chan, Jacob W. Cooper, Robert Hancock, Adam Kabela, Ander Lamaison, Taísa Martins, Roberto Parente, Samuel Mohr, Jonathan A. Noel, Sergey Norin, Péter Pál Pach, Yanitsa Pehova, Oleg Pikhurko, Maryam Sharifzadeh, Fiona Skerman, Jan Volec and Fan Wei.

V Shades of Perfection

János Pach

Rényi Institute, Budapest & IST Austria

In the early period of development of graph theory, perfect graphs played a central role. They motivated a lot of research in optimization, graph coloring, theory of algorithms, and in hypergraph theory. However, most graphs are not perfect. Inspired by pioneering work of Asplund and Grunbaum, at a conference in Pokrzywna (Poland), Gyárfás initiated the systematic study of "nearly perfect" graphs. These are infinite classes of graphs with the property that their chromatic numbers are bounded by a function f of their clique numbers. How far these graphs are from being perfect, depends on the growth rate of f. It has turned out that many "natural", geometrically defined classes of graphs are nearly perfect, but there are also interesting exceptions. The problem is related to the celebrated Erdős-Hajnal conjecture, one of the most challenging open problems in Ramsey theory.

VI Breaking symmetry with two colors

Thomas Tucker

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Given a group A acting on a set X, an assignment of colors to the elements of X is *distinguishing* if the only element of A preserving the coloring fixes all $x \in X$. The least number of colors needed is the *distinguishing number* $D(A, X)$. If $A = Aut(G), X = V(G)$ for the graph G, then $D(A, X) = D(G)$ is called the distinguishing number (or asymmetric coloring number $ACN(G)$); if instead we use $X = E(G)$, we have the distinguishing index $D'(G)$. Albertson and Collins (1996) investigated the activity on distinguishing graphs, but, unbeknownst to them, the idea extends back to Babai (1977) for trees and Gluck(1983), Cameron et al (1984) for finite permutation groups. This talk surveys the principal results on distinguishing graphs and maps, focussing on classification of situations where $D(A, X) = 2$. Generally, if every non-identity $a \in A$ moves enough $x \in X$, then $D(A, X) = 2$. For locally finite, infinite graphs it was conjectured 10 years ago that "enough" is infinity. This Infinite Motion Conjecture has been proved in the last year by Babai.

1. Induced colorings of graphs and digraphs

M. Anholcer[(1)], B. Bosek[(2)], J. Grytczuk[(3)]

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[(2)] Jagiellonian University, Kraków, Poland
[(3)] Warsaw University of Technology, Warszawa, Poland

Let $G = (V, E)$ be a graph. A labeling $\varphi : V \cup E \to \mathbb{N} = \{1, 2, \dots\}$ is called a (K, L)-*total labeling*, if $\varphi(x) \leqslant K$ and $\varphi(uv) \leqslant L$, for all $x \in V$ and $uv \in E$. For every vertex $x \in V$, we define the *weight* of x by $w_\varphi(x) = \varphi(x) + \sum_{v \in N(x)} \varphi(xv)$. One may think of the function $w_\varphi : V \to \mathbb{N}$ obtained in this way, as of a vertex coloring *induced* by the labeling φ.

We consider the following general question: *What type of a graph coloring can be realized as an induced coloring with labels of bounded size?* In a positive case, we look for the least possible constants in a (K, L)-total labeling inducing a desired coloring of any graph. For example, it is known that every graph has a (K, L)-total labeling inducing the usual proper coloring with $(K, L) = (1, 5)$ and $(K, L) = (2, 3)$. It is conjectured that the same is true with $(K, L) = (1, 3)$ and $(K, L) = (2, 2)$ (these are the famous *1-2-3 Conjecture* and *1-2 Conjecture*, respectively). On the other hand, it is known that 2-*distance coloring* cannot be induced by a $(1, L)$-total labeling with any constant L.

In the talk we shall discuss the above "inducibility" issues for *majority*, *acyclic*, *star*, and *nonrepetitive* coloring of graphs, in both, directed and undirected version.

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2. From domination to isolation of graphs

Peter Borg

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In 2017, Caro and Hansberg [5] introduced the isolation problem, which generalizes the domination problem. Given a graph G and a set $\mathcal{F}$ of graphs, the *$\mathcal{F}$-isolation number of G* is the size of a smallest subset D of the vertex set of G such that the graph obtained from G by removing the closed neighbourhood of D does not contain a copy of a graph in $\mathcal{F}$. When $\mathcal{F}$ consists of a 1-clique, the $\mathcal{F}$-isolation number is the domination number. Caro and Hansberg [5] obtained many results on the $\mathcal{F}$-isolation number, and they asked for the best possible upper bound on the $\mathcal{F}$-isolation number for the case where $\mathcal{F}$ consists of a k-clique and for the case where $\mathcal{F}$ is the set of cycles. The solutions [1, 3] to these problems will be presented together with other results, including an extension of Chvátal's Art Gallery Theorem. Some of this work was done jointly with Kurt Fenech and Pawaton Kaemawichanurat.

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3. On consecutive colouring and re-consecutiveness of oriented graphs

M. Borowiecka-Olszewska[(1)], E. Drgas-Burchardt[(1)], R. Zuazua[(2)]

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[(2)] National Autonomous University of Mexico, Mexico

We consider an arc colouring of oriented graphs such that for each vertex the colours of all out-arcs incident with the vertex and the colours of all in-arcs incident with the vertex form intervals. Oriented graphs having such the colouring are called consecutively colourable. We analyze the parameter $rc(D)$ which denotes the minimum number of arcs of D that should be reversed so that a resulting oriented graph is consecutively colourable. We prove that for each non-negative integer p there exists an oriented graph D with the property $rc(D) \geqslant p$. Next, we show an upper bound on $rc(D)$ for some classes of oriented graphs D.

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4. Upper bound on the domination number of graphs with minimum degree four

Csilla Bujtás

University of Pannonia, Veszprém, Hungary

In the talk, we prove that if G is a connected graph of order n and with minimum degree 4, then its domination number $\gamma(G)$ satisfies $\gamma(G) \leqslant \frac{71n+5}{200}$. Moreover, $\gamma(G) \leqslant \frac{71n}{200}$ also holds under the same conditions, if n is large enough. It improves the best known upper bound to date which was established by Sohn and Yuan [4] in 2009. We also discuss recent results from [1] and [3] on the domination number of graphs with minimum degree 5 and 6 respectively.

References

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5. Structural properties of essentially-highly-connected polyhedral graphs

K. Čekanová, T. Madaras

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A k-connected graph is called essentially $(k+1)$-connected if each its vertex k-cut leaves at most one nontrivial component. We explore the local structure of essentially 4- and 5-connected plane graphs, focusing on existence of small clusters of faces of small sizes as well as the small subgraphs (or sets of subgraphs) having vertices of degrees upper bounded by small constants; as an application, we show that the cyclic edge connectivity of essentially 5-connected plane graphs is finite.

6. Irregular labeling of digraphs

Sylwia Cichacz

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Let $\overrightarrow{G}$ be a directed graph of order n with no component of order less than 4, and let Γ be a finite Abelian group such that $|\Gamma| \geqslant n+6$. We show that there exists a mapping ψ from the arc set $E(\overrightarrow{G})$ of $\overrightarrow{G}$ to an Abelian group Γ such that if we define a mapping φ_ψ from the vertex set $V(\overrightarrow{G})$ of $\overrightarrow{G}$ to Γ by

$$\varphi_\psi(x) = \sum_{y \in N^+(x)} \psi(xy) - \sum_{y \in N^-(x)} \psi(yx), \quad (x \in V(\overrightarrow{G})),$$

then φ_ψ is injective.

7. Independent Domination Subdivision in Graphs

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A set S of vertices in a graph G is a dominating set if every vertex not in S is adjacent to a vertex in S. If, in addition, S is an independent set, then S is an independent dominating set. The domination (independent domination) number $\gamma(G)$ ($i(G)$, resp.) of G is the minimum cardinality of a dominating (an independent dominating, resp.) set in G. The domination (independent domination) subdivision number $\mathrm{sd}_\gamma(G)$ ($\mathrm{sd}_i(G)$, resp.) is the minimum number of edges that must be subdivided (each edge in G can be subdivided at most once) in order to increase the domination (independent domination, resp.) number. We show [1] that for every connected graph G on at least three vertices, the parameter $\mathrm{sd}_i(G)$ is well defined and differs significantly from the well-studied domination subdivision number $\mathrm{sd}_\gamma(G)$. For example, if G is a block graph, then $\mathrm{sd}_\gamma(G) \leqslant 3$, while $\mathrm{sd}_i(G)$ can be arbitrary large. Further we show that there exist connected graph G with arbitrarily large maximum degree $\Delta(G)$ such that $\mathrm{sd}_i(G) \geqslant 3\Delta(G) - 2$, in contrast to the known result that $\mathrm{sd}_\gamma(G) \leqslant 2\Delta(G) - 1$ always holds. Among other results, we present a simple characterization of trees T with $\mathrm{sd}_i(T) = 1$.

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8. Fast winning strategies for Staller in the Maker-Breaker domination game

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The Maker-Breaker domination game is played on a graph G by two players, called Dominator and Staller, who alternately choose a vertex that was not played so far. Dominator wins the game if he forms a dominating set. Staller wins if she claims all vertices from a closed neighborhood of a vertex $v \in V(G)$. In this talk, we will discuss possible optimal strategies for Staller when she can win the game. We will introduce the invariant $\gamma'_{\rm SMB}(G)$ (resp., $\gamma_{\rm SMB}(G)$) which is the smallest integer k such that, under any strategy of Dominator, Staller can win the game by playing at most k vertices, if Staller (resp., Dominator) starts the game.

We will prove some basic properties of $\gamma_{\rm SMB}(G)$ and $\gamma'_{\rm SMB}(G)$ and study the effect to parameters under some elementary operators. We will also present graphs with small winning numbers and consider the game on some classes of graphs, in particular on subclasses of trees.

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9. On the chromatic edge stability index of graphs

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Given a non-trivial graph G, the minimum cardinality of a set of edges F in G such that $\chi'(G \setminus F) < \chi'(G)$ is called the chromatic edge stability index of G, denoted by $es_{\chi'}(G)$, and such a (smallest) set F is called a (minimum) mitigating set. In this talk we investigate graphs with extremal and near-extremal values of $es_{\chi'}(G)$. We establish that the odd cycles and K_2 are exactly the regular connected graphs with the chromatic edge stability index 1; on the other hand, we present that it is NP-hard to verify whether a graph G has $es_{\chi'}(G) = 1$. We also prove that every minimum mitigating set of an r-regular graph G, where $r \neq 4$, with $es_{\chi'}(G) = 2$ is a matching. Furthermore, we propose a conjecture that for every graph G there exists a minimum mitigating set, which is a matching, and prove that the conjecture holds for graphs G with $es_{\chi'}(G) \in \{1, 2, \lfloor n/2 \rfloor - 1, \lfloor n/2 \rfloor\}$, and for bipartite graphs.

10. Combinatorial spectra of graphs

M. Dzúrik

Masaryk University, Brno, Czech republic

In the article [1] we have created a generalization of the Hamiltonian spectrum of a graph G called the H-Hamiltonian spectrum of the graph G denoted by $\mathscr{H}_H(G)$. Not only does this generalization give us the opportunity to talk, for example, about the isomorphism of graphs and the regularity of graphs in the language of these spectra, but there are several relations between $\mathscr{H}_H(G)$ and $\mathscr{H}_{H'}(G)$ for related H and H', for example for $H' = \overline{H}$. And so this brings some basic calculus to this area.

This approach has led us to one more generalization, which we call the combinatorial spectrum. This spectrum is now for R-weighted graphs, more precisely for sets of R-weighted graphs and we denote it $\mathscr{H} * \mathscr{G}$. This will play role of multiplication, we also get some addition $+$. When we denote $\mathscr{P}(Graphs)$ the set of all sets of graphs we will get that $(\mathscr{P}(Graphs), *, +)$ is a R-module, semigroup (or comutative monoid) and something close to ring. The most important thing is that most of the basic concepts of graph theory, such as maximum pairing, vertex and edge connectivity and coloring, Ramsey numbers, isomorphisms and regularity, can be expressed in the language of these combinatorial spectra. Together with the already mentioned calculus, it gives hope that all these concepts could be linked, studied together and transfer the results from one to the other.

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11. Distinguishing index of the graphs with $\delta \geqslant \frac{\Delta}{2}$.

Mariusz Grech, Andrzej Kisielewicz

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Let G be a connected, finite or infinite, graph. An edge-coloring $\phi : E(G) \to \{1, 2, ..., r\}$ of a graph G is said to be *asymmetric* if no nontrivial automorphism of G preserves colors of the edges.

The minimal possible r is called the *distinguishing index* of G and denoted by $D'(G)$.

In [1] F. Lehner, M. Pilśniak, M. Stawiski proved that that for connected regular graphs other than K_2, $D'(G) \leqslant 3$.

Here we extend this result to a much larger class of connected graphs satisfying the condition $\delta \geqslant \frac{\Delta}{2}$.

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12. Maximal number of cycles of a given length in oriented graphs

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[4] Czech Technical University, Prague, Czech Republic

We will discuss the problem of finding the maximal possible number of directed cycles of a given length in oriented graphs.

13. On a Problem of Steinhaus

M.Anholcer(1), B.Bosek(2), J.Grytczuk(3), G.Gutowski(2), J.Przybyło(4), R.Pyzik(2), M.Zając(3),

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(2) Jagiellonian University
(3) Warsaw University of Technology
(4) AGH University of Science and Technology

In this talk, inspired by the *17-points Problem* of Steinhaus (Problems 6 and 7 from his famous book *Sto zadań*), we discuss infinite sequences of real numbers in $[0,1)$. For a function $f : \mathbb{N} \to \mathbb{N}$, we say that a sequence X is f-piercing if for every integer $m \geqslant 1$, the first $f(m)$ elements of X contain at least one element in every interval $\left[\frac{i}{m}, \frac{i+1}{m}\right)$ for every $i = 0, 1, \ldots, m-1$. There is a nice construction of an $\left(\frac{m}{\ln 2}\right)$-piercing sequence due to de Bruijn and Erdős which satisfies even stronger piercing properties. We are able to show that this is best possible, as there are no $(\alpha m + o(m))$-piercing sequences for $\alpha < \frac{1}{\ln 2}$. Our results allow for some new tight linear bounds for similar concepts defined for finite sequences. Ideas presented during this talk are described in full detail in our arXiv manuscript [1].

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14. Generalized Turán number of double stars and other bipartite graphs in triangle-free graphs

Ervin Győri

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In a generalized Turán problem, two graphs H and F are given and the question is the maximum number of copies of H in an F-free graph of order n. In the first part of the talk, , we study the number of double stars $S_{k,l}$ in triangle-free graphs. We also study an opposite version of this question: what is the maximum number of edges and triangles in graphs with double star type restrictions, which leads us to study two questions related to the extremal number of triangles or edges in graphs with degree-sum constraints over adjacent or non-adjacent vertices.

In the second part of the paper, the double stars are replaced by arbitrary bipartite graphs. But first, a more general conjecture.

Conjecture. (Lidický, Murphy) Let G be a graph and let $r > \chi(G)$ be an integer. Then there exist integers $n_1, n_2, \ldots, n_{r-1}$ such that $n_1 + n_2 + \cdots + n_{r-1} = n$ and we have

$$ex(n, G, K_r) = G(K_{n_1, n_2, \ldots, n_{r-1}}).$$

Actually, there is a counterexample to the Conjecture for every $r \geqslant 3$. The case $r = 3$ is specially interesting. We found some extra conditions what make the conjecture true. But the exact condition is still not clear.

The talk is based on a paper joint with R. Wang, S. Woolfson, and another one joint with A. Grzesik, N. Salia, C. Tompkins

15. 2-nearly Platonic graphs

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A 2-nearly Platonic graph of type (k, d) is a k-regular plane graph with f faces, $f - 2$ of which are of size d and the remaining two are of sizes d_1, d_2, both different from d. Such a graph is called balanced if $d_1 = d_2$. We show that all connected 2-nearly Platonic graphs are balanced. This proves a recent conjecture by Keith, Froncek, and Kreher. We also show that any 2-nearly Platonic graph belongs to one of 15 well defined infinite classes. The latter states more precisely the statement of Deza, Dutour Sikirič, and Shtogrin from 2013, and of Froncek. Khorsandi, Musawi, and Qui from 2021 that there are only 14 such classes. Moreover, our short proof provides a complete characterization of all 2-nearly Platonic graphs.

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16. Ramsey numbers for families of graphs

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The Ramsey number $R_k(G)$ is the smallest n such that for every colouring of the edges of a complete graph on n vertices with k colors there is a monochromatic copy of G.

After Aharoni *et al.* [1], we consider a generalization of Ramsey numbers for families of graphs. For a family of graphs $\mathcal{G}$ by $R_k(\mathcal{G})$ we denote the smallest n such that for every colouring of a complete graph on n vertices with k colors there is a monochromatic copy a graph $G \in \mathcal{G}$. We determine the Ramsey numbers for family of cycles, family of odd cycles. Moreover we give some bounds on the Ramsey number for family of even cycles, family of cycles with restrictions on lenghts, family of stars and matchings.

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17. Graphs with a unique maximum independent set up to automorphisms

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Let G be a graph and let $S \subset V(G)$ has some property $\mathcal{P}$. By $\rho(G)$ we denote maximum (respectively minimum) cardinality of a subset of $V(G)$ with property $\mathcal{P}$. We say that G is ρ-unique if there is exactly one set $S \subset V(G)$ of cardinality $\rho(G)$ with property $\mathcal{P}$. Graphs with unique maximum independent set have been studied, inter alia, in [2] and [3].

A graph G is called ρ-iso-unique if for any two subsets $S_1, S_2 \subset V(G)$ of cardinality $\rho(G)$ there is an automorphism $\varphi \in \mathrm{Aut}(G)$, such that $\varphi(S_1) = S_2$. In [1], we start the investigation into α-iso-unique graphs by giving the characterization of such trees and partially generalizing the results on chordal graphs. We state some results about the problem complexity and some results concerning α-iso-unique cartesian products.

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18. On minimum intersections on certain secondary dominating sets

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Let $k \geqslant 1$ be an integer. A subset $D \subset V(G)$ is (1,k)-dominating if for every vertex $v \in V(G) \setminus D$ there are $u, w \in D$ such that $uv \in E(G)$ and $d_G(v, w) \leqslant k$. If $k = 1$ then we obtain the definition of (1,1)-dominating sets, which are also known as 2-dominating sets. If $k = 2$ then we have the concept of (1,2)-dominating sets, see [1]. A *proper* (1,2)-*dominating set* is (1,2)-dominating set that is not (1,1)-dominating, see [3]. Even though (1,1)-dominating sets and proper (1,2)-dominating sets cannot be equal, they do not have to be disjoint. Therefore, it is natural to ask what is the minimum possible cardinality of the intersection of such sets in a given graph.

In the talk we present some results concerning minimum intersections of the (1,1)-dominating sets and proper (1,2)-dominating sets in some classes of graphs.

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19. On local antimagic chromatic number of graphs with cut-vertices

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An edge labeling of a connected graph $G = (V, E)$ is said to be local antimagic if it is a bijection $f : E \to \{1, \ldots, |E|\}$ such that for any pair of adjacent vertices x and y, $f^+(x) \neq f^+(y)$, where the induced vertex label $f^+(x) = \sum f(e)$, with e ranging over all the edges incident to x. The local antimagic chromatic number of G, denoted by $\chi_{la}(G)$, is the minimum number of distinct induced vertex labels over all local antimagic labelings of G. In this paper, the sharp lower bound of the local antimagic chromatic number of a graph with cut-vertices given by pendants is obtained. The exact value of the local antimagic chromatic number of many families of graphs with cut-vertices (possibly given by pendant edges) are also determined. Consequently, we partially answered Problem 3.1 in [Local antimagic vertex coloring of a graph, *Graphs and Combin.*, **33** (2017), 275–285.].

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20. The longest CT-paths in 4-regular plane graphs

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Let G be a 4-regular graph with prescribed rotation system and let e_1, e_2, e_3, e_4 be edges incident with a vertex v in that order. The pairs e_1, e_3 and e_2, e_4 are called CT-adjacent in G. A CT-path (CT-trail) is a path (trail) in which every two consecutive edges are CT-adjacent. Simple 4-regular plane graphs consisting of a single closed CT-trail are called knots; if every closed CT-trail of a simple 4-regular plane graph is a CT-cycle, then the graph is called Grötzsch-Sachs graph.

In this talk, we show that the longest CT-path in an n-vertex knot has at most $n-2$ vertices, and give construction of knot with longest CT-path with that number of vertices for every $n \geqslant 8$; also we prove that the longest CT-path in an n-vertex Grötzsch-Sachs graph has at most $\frac{2n}{3}$ vertices. Next, we show that there exists infinitely many simple 4-regular plane graphs whose longest CT-paths contain just eight vertices; we conjecture that, apart of the single exception, all graphs with longest 8-vertex paths are Grötzsch-Sachs graphs. In addition, we provide an analogous construction yielding knots with longest 16-vertex paths. In the case when the longest CT-path has less than eight vertices, we pose a conjecture (supported by computer simulations generating the list of feasible graphs) that there is only finitely many corresponding 4-regular plane graphs; we have confirmed its validity for longest CT-paths on four and five vertices.

21. Homology of graphs and path complexes

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A *path complex* P on a finite set V is a collection of finite sequences of points $i_0 \dots i_n$ from V such that if path v belongs to P then truncated paths $i_1 \dots i_n$ and $i_0 \dots i_{n-1}$ are also in P [1]. Any poset $(V, \leqslant)$ determines naturally a path complex P with paths that are given by sequences $i_0 \dots i_n$ with $i_{k-1} < i_k$ for $k = 1, \dots, n$. Any simplicial complex S determines naturally a path complex $P(S)$ consisting of the sequences of simplexes $\sigma_0 \dots \sigma_n$ such that $\sigma_{k-1} \subsetneqq \sigma_k$. However, the main motivation for considering path complexes comes from the graph theory. Any (di)graph naturally defines a path complex where allowed paths go along edges (arrows). For any path complex P we can define a *path homology groups*. It follows from this definition that the simplicial homology is a particular case of the path homology. We present the path homology theory for various categories of graphs and describe its relations to Eilenberg-Steenrod axiomatic in the classical algebraic topology [1]. We give examples of application to colored digraphs and graphs [2, 3].

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22. Minimal host graph

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For a given hypergraph H a *host graph* (or *support*) G is a graph spanned on the same set of vertices as H such that each hyperedge of H induces a connected subgraph in G [1]. A concept of host graph is often used to define hypergraph classes like hypertrees or hypercycles. Herein, we would like to explore a problem of finding minimal (with respect to number of edges) host graphs for a given hypergraph H.

We consider various classes of hypergraphs and discuss complexity status of the problem. Note that, finding minimal host graph is trivial for linear hypergraphs. However, it is NP-complete for simple hypergraphs.

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23. M_f-edge colorings of graphs

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For a graph G and a function f which assigns positive integers to vertices of G, an M_f-edge coloring is an edge coloring of G in which, for each vertex v, the number of colors used on edges incident to v is at most $f(v)$. The maximum number of colors of an M_f-edge coloring of G is denoted by $\mathcal{K}_f(G)$. It was proved in [1] that the problem of computing $\mathcal{K}_f(G)$ is NP-hard; it is NP-hard even in the case when $f(v) = 2$ for each vertex v. We present several bounds on $\mathcal{K}_f(G)$ and graphs attaining these bounds. As a consequence, we determine exact values of $\mathcal{K}_f(G)$ for graphs of particular classes, such as trees, complete graphs, complete multipartite graphs etc. We also present the algorithm which computes $\mathcal{K}_f(G)$ of a cactus graph G in quadratic time with respect to the order of G.

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24. On Fibonacci numbers in edge coloured unicyclic graphs

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Let G be an undirected, connected graph. Let $\mathcal{C} = \{A, B\}$ be the set of two colours. We say that a graph G is an $(A, 2B)$-edge coloured if for every maximal B-monochromatic subgraph $H \subseteq G$ there is a partition of H into edge disjoint paths of the length 2. In the talk we present the lower bound and the upper bound for the number of all $(A, 2B)$-edge colourings in unicyclic graphs. We give full characteristic of graphs achieving these extreme values. Moreover, we determined the successive extreme unicyclic graphs with respect to the number of all $(A, 2B)$-edge colourings.

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25. Twins in various settings

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Generally speaking, twins in a combinatorial structure are defined as two disjoint, isomorphic substructures. In my talk I will present recent results concerning the size of the largest twins guaranteed in every permutation, ordered matching, and a finite word. Random counterparts of each problem will also be discussed.

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26. The maximum number of $K_{s,s}$ copies in C_{2s+2}-free graphs

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For every integers $s \geqslant 3$ and sufficiently large n we determine the maximum number of copies of $K_{s,s}$ in an n-vertex C_{2s+2}-free graph.

27. Light 3-stars in embedded graphs

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For integers $k \geqslant 1$ and $1 \leqslant t \leqslant 3$, let $g(k,t)$ be the minimum integer such that every graph with girth at least $g(k,t)$, minimum degree at least 2 and no $(k+1)$-path consisting of vertices of degree 2, has a 3-vertex with at least t neighbors of degree 2. For the class of plane graphs there are many results concerning existence of a 3-vertex with specified number of 2-neighbors. Recently, Borodin and Ivanova established the value of $g(k,t)$ for all combinations of k and t (where $k \geqslant 1$ and $t \in \{1,2,3\}$). In the talk we present how the situation changes for the class of graphs embedded on a surface(s) with non-positive Euler characteristic.

28. The Neighbour Sum Distinguishing Relaxed Edge Colouring

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A k-edge colouring of a graph with colours in $\{1, 2, \ldots, k\}$ is neighbour sum distinguishing if, for any two adjacent vertices, the sums of the colours of the edges incident with each of them are distinct. It can easily be observed that every connected graph different from K_2 admits neighbour sum distinguishing edge coloring. A graph is nice if it has no components isomorphic to K_2. We consider the neighbour sum distinguishing edge colouring in which each monochromatic set of edges induces a subgraph with maximum degree at most d. We call such an edge colouring a neighbour sum distinguishing d-relaxed k-edge colouring. We denote by $\chi'^{d}_{\sum}(G)$ the smallest value of k such that such a colouring of G exists. Note that, for $d = 1$ and $d = \Delta(G)$, we obtain the neighbour sum distinguishing edge coloring related with two famous conjectures: the 1-2-3 Conjecture states that $\chi^{\Delta(G)}_{\sum}(G) \leqslant 3$ for any nice graph G ([2]); and the other states that $\chi^{1}_{\sum}(G) \leqslant \Delta(G) + 2$ for any nice graph $G \neq C_5$ ([1]).

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29. The Petersen and Heawood graphs make up graphical twins via induced matchings

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Inspired by the Isaacs remark (published in 1975), we show that the Petersen and Heawood graphs (P_g and H_g) make up a bijectively linked pair of graphs. Another related new result is that P_g is uniqely decomposable into five induced 3-machings. It shows a kind of the structural rigidity of Pg. Information on maximal matchings with sizes 3,4 and 5 in P_g is recalled. Constructive proofs confirm that the strong chromatic index $sq(P_g) = 5$ and $sq(H_g) = 7$. The three numerical edge coloring partitions for P_g are also determined.

30. Light edges in embedded graphs with minimum degree 2

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The weight of an edge e is the degree-sum of its end-vertices. An edge $e = uv$ is of type (i, j) if $deg(u) \leqslant i$ and $deg(v) \leqslant j$. Kotzig proved that every 3-connected plane graph contains an edge of weight at most 13. Ivančo described bounds for weights of edges in the class of graphs embeddable on the orientable surfaces with higher genus. Jendroľ and Tuhársky investigated the weight of edges in the class of graphs embeddable on the nonorientable surfaces with higher genus. Later Jendroľ, Tuhársky and Voss described exact types of edges in large embedded maps with minimum degree 3.

In the talk we consider connected graphs on n vertices, minimum degree two, minimum face degree ρ, embedded on a surface $\mathcal{S}$ with non-positive Euler characteristic. We can prove that every such graph contains an edge of type

- $(2, \infty)$, $(3, 12)$, $(4, 8)$ or $(6, 6)$ if $\rho = 3$ and $n > 24|\chi(\mathcal{S})|$,
- $(2, \infty)$, $(3, 6)$ or $(4, 4)$ if $\rho = 4$ and $n > 20|\chi(\mathcal{S})|$,
- $(2, 6)$ or $(3, 3)$ if $\rho \in \{5, 6\}$ and $n > 27|\chi(\mathcal{S})|$,
- $(2, 4)$ if $\rho \in \{7, 8\}$ and $n > 14|\chi(\mathcal{S})|$,
- $(2, 3)$ if $\rho \in \{9, 10, 11, 12\}$ and $n > 15|\chi(\mathcal{S})|$,
- $(2, 2)$ if $\rho \geqslant 13$ and $n > 35|\chi(\mathcal{S})|$.

We will also discuss the quality of our results.

31. Distinguishing regular graphs

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Call an edge colouring of a graph G distinguishing if the only automorphism of G that preserves the colouring is the identity. The distinguishing index of graph G is the least number of colours in a distinguishing edge colouring of G, and it is denoted by $D'(G)$. Lehner, Pilśniak and Stawiski proved that the distinguishing index of every locally finite (infinite or finite) connected regular graph except K_2 satisfies $D'(G) \leqslant 3$. Grech and Kisielewicz extended this result to locally finite connected graphs for which the minimum degree is at least half the maximum degree. Lehner, Pilśniak and Stawiski conjectured that $D'(G) \leqslant 2$ for every locally finite connected regular graph on at least seven vertices. We prove this conjecture.

Pilśniak proved that the distinguishing index of finite traceable graph of order at least 7 is at most 2. It follows from our result that vertex-transitive graphs satisfy the same bound. Therefore, this may be seen as a support for the well-known Lovász Conjecture stating that every finite connected vertex transitive graph is traceable.

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32. On graphs with prescribed neighborhood degrees of vertices

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Let $G = (V, E)$ be a symmetric graph with n vertices and let $\deg(v)$ denote the degree of vertex v in G. We define **a neighborhood degree** of a vertex v, denoted by n-degree(v), as the sum of the degrees of its neighbors.

Suppose we are given a set of n positive integers $\mathbf{d}$. There are several conditions and algorithms which can be used to decide whether $\mathbf{d}$ is a degree sequence of a graph. We show however that it is NP-hard to decide whether $\mathbf{d}$ is an n-degree sequence of a graph even in the class of caterpillars. We present also a polynomial time algorithm for a special case of the problem.

33. Ramsey numbers of Boolean lattices

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The *poset Ramsey number* $R(Q_m, Q_n)$ is the smallest integer N such that any blue-red coloring of the elements of the Boolean lattice Q_N has a blue induced copy of Q_m or a red induced copy of Q_n. The *weak poset Ramsey number* $R_w(Q_m, Q_n)$ is defined analogously, with weak copies instead of induced copies.

Axenovich and Walzer[1] showed that $n + 2 \leqslant R(Q_2, Q_n) \leqslant 2n + 2$. Recently, Lu and Thompson[5] improved the upper bound to $\frac{5}{3}n + 2$. We solve this problem asymptotically by showing that $R(Q_2, Q_n) = n + O(n/\log n)$. Recent work of Axenovich and Winter[2] implies that the $n/\log n$ term is required.

In the diagonal case, Cox and Stolee[4] proved $R_w(Q_n, Q_n) \geqslant 2n + 1$ using a probabilistic construction. In the induced case, Bohman and Peng[3] showed $R(Q_n, Q_n) \geqslant 2n + 1$ using an explicit construction. Improving these results, we show that $R_w(Q_m, Q_n) \geqslant n + m + 1$ for all $m \geqslant 2$ and large n by giving an explicit construction.

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34. Cycle decompositions of complete 3-uniform hypergraphs

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A 3-uniform cycle of length k has k vertices $v_1, \ldots, v_k$ and k edges $v_1v_2v_3$, $v_2v_3v_4, \ldots, v_{k-2}v_{k-1}v_k, v_{k-1}v_kv_1, v_kv_1v_2$. We discuss constructions and open problems on edge decompositions of complete 3-uniform hypergraphs into cycles of given length, mostly in the case of $k = 5$.

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35. Distance Fibonacci polynomials in a graph, combinatorial and matrix perspective

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The classical Fibonacci polynomials defined by the recurrence relation

$$f_n(x) = xf_{n-1}(x) + f_{n-2}(x), \quad n \geqslant 2,$$

with initial conditions $f_0(x) = 0, f_1(x) = 1$ were introduced by the Belgian mathematician E.C. Catalan in 1883 and have been intensively studied since then. The interest in these polynomials has contributed to the emergence of many generalizations. In the talk we present a generalization given by the following recursion

$$f_n(k, x) = xf_{n-1}(k, x) + f_{n-k}(k, x), \quad n \geqslant k,$$

with initial conditions $f_n(k, x) = x^n$ for $n = 0, 1, \dots k - 1,\ k \geqslant 2$. We focus on a graph interpretation of these polynomials, their connections with Pascal's triangle and relations with Hessenberg matrices.

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36. Adjacent Vertex Distinguishing Total Coloring of Corona Product of Graphs

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An adjacent vertex distinguishing total k-coloring f of a graph G is a proper total k-coloring of G such that no pair of adjacent vertices has the same color sets, where the color set at a vertex v, $C_f^G(v)$, is $\{f(v)\} \cup \{f(vu) | u \in V(G), vu \in E(G)\}$.

In 2005 Zhang et al. posted the conjecture (AVDTCC) that every simple graph G has adjacent vertex distinguishing total $(\Delta(G)+3)$-coloring.

In this talk we consider adjacent vertex distinguishing total k-coloring of many coronas, in particular for generalized, simple and l-coronas of graphs.

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37. Constructing sparsest ℓ-hamiltonian saturated k-uniform hypergraphs for a wide range of ℓ

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Given $k \geqslant 3$ and $1 \leqslant \ell < k$, an (ℓ, k)-cycle is one in which consecutive edges, each of size k, overlap in exactly ℓ vertices. We study the smallest number of edges in k-uniform n-vertex hypergraphs which do not contain hamiltonian (ℓ, k)-cycles, but once a new edge is added, such a cycle is promptly created. It has been conjectured [1, 2] that this number is of order n^ℓ and confirmed [2, 3] for $\ell \in \{1, k/2, k-1\}$, as well as for the upper range $0.8k \leqslant \ell \leqslant k-1$. Here we extend the validity of this conjecture to the lower-middle range $(k-1)/3 \leqslant \ell < k/2$.

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